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Multi-objective Evolutionary Algorithms to Cope with Single-objective Optimization and Post-optimal Evolution for Scalarlized Problems

Yoshiaki SHIMIZU*
* Optimization Engineering Laboratory
4-10-90 Fuseya-cho, Izumi, Osaka, 594-0031, Japan

Abstract
To cope with many difficult problems that must be solved urgently for sustainable development, practical approaches available for rational decision making is highly demanded in modern technologies. In such situation, various optimization methods have been successfully applied so far. In advance, under multiple goals some of which conflict with each other, a particular field known as multi-objective optimization has been studied from various aspects. Thereat, multi-objective evolutionary algorithms (MOEA) are especially interested in these decades. They are viewed as a useful technique for revealing a wide relation of trade-off among the conflicting objectives and supporting multi-objective optimization that will attempt to obtain a unique preferntially optimal solution. To enhance its availability, in this paper, we have proposed a simple procedure for solving single-objective optimization problems (MOPs) using MOEA. Then, the idea is extended to solve the scalarlized MOP such as weighing and ϵ-constraint (lexicographic) formulations. Being classic, they are often used in various situations even presently due to the effectiveness regardless of their simplicity. Moreover, we propose to carry out a post-optimal evolution for repairing some shortcomings inherent to those approaches and making them adaptive. Actually, it is deployed in co-operation with our elite-induced evolutionary algorithms. After preliminarily examining a few properties of the idea, a set of benchmark problems including multiple peaks have been solved to examine the performance as a global optimization technique. The significance of the post-optimal evolution for the scalarlized MOP has been also verified numerically.

Keywords : optimization, multi-objective evolutionary method, NSGA-II, post-optimal evolution, elite-induced evolutionary algorithm, scalarlized multi-objective optimization method

1. Introduction

To cope with many difficult problems that must be solved urgently for sustainable development, practical optimization methods are highly demanded for supporting rational decision-making in modern technologies. In this sense, meta-heuristic optimization methods opened a new horizon since they can work with various situations flexibly and effectively. They never need differential information of functions at all, go well with meta-model or model of model and achieve global optimization. Noticing the amazing progress of simulation technique as in software and computer as in hardware, such feature is quite suitable for practical optimization. Moreover, such approach has successfully extended to the area associated with multi-objective optimization. Actually, multi-objective evolutionary algorithm (MOEA) has been greatly studied in these decades (Coello, 2001)and is still developing under various interests in future direction (Coello, 2012; Lucken et al., 2014). These MOEAs aim at revealing a wide relation on objective function values among the conflicting objectives and supporting multi-objective optimization that tries to obtain a unique preferentially optimal solution.

To enhance availability of such MOEA, in this paper, we have proposed a simple procedure for solving single-objective optimization problem (SOP) using MOEA. Besides such a plain application, it is straightforwardly extended to solve certain scalarlized MOPs given by the weighing and ϵ-constraint formulations, for example. Being classic, they are often used even presently due to the effectiveness regardless of their simplicity. It is interesting to show the proposed procedure can derive the unique preferentially optimal solution of such scalarlized MOPs by using the method for multi-objective analysis such as MOEA. So far, such ideas have not been reported anywhere.

Moreover, another idea is deployed as a post-optimal evolution for repairing some shortcomings inherent to those classic approaches. Such concern is involved in one of the future challenges referred to incorporation of DM's preference (Coello, 2012) and has been discussed only in a few studies (Karahan and Koksalan, 2010; Branke, 2008). Actually, it is developed in co-operation with our elite-induced EAs. It should be noted here we can conveniently use the same MOEA both for the prior optimization and the post-optimal evolution. Eventually, the aims of this study is to extend the availability of the conventional MOEAs and provide a rational procedure to repair an inherent shortcoming embedded in when solving the scalarlized MOPs.

The rest of this chapter is organized as follows. In Chapter 2, Concerns associated with the present studies are described briefly. Chapter 3 describes the proposed idea and its cool application associated with the post-optimal evolution. In Chapter 4, after a preliminary numerical experiment, solution ability for SOP is examined through various benchmark problems. Then, significance of the post-optimal evolution is discussed demonstratively. Some conclusions are given in Chapter 5.

2. Problem Statements

2.1. Single-objective and multi-objective optimizations

Generally, SOP is formulated as [Problem 1].

$$\text{[Problem1]}\ \ \min f(\boldsymbol{x}) \text{ subject to } \boldsymbol{x} \in X$$

where $\boldsymbol{x}$ denotes a decision variable vector, X a feasible region and f a scalar objective function.

Though many mathematical programming methods have been traditionally applied, in modern optimization, they are likely replaced with meta-heuristic or EAs. This is because they can practically cope with various simulation models and expect to obtain a global solution even for complicated problems.

Meanwhile, MOP is formulated as [Problem 2].

$$\begin{aligned}\text{[Problem2]}\ \ &\min \boldsymbol{f}(\boldsymbol{x}) = \{f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_N(\boldsymbol{x})\}\\ &\text{subject to } \boldsymbol{x} \in X\end{aligned}$$

where $\boldsymbol{f}$ denotes a vector objective function some elements of which conflict with one another.

The aim of MOP is to obtain a unique preferentially optimal solution through subjective judgments of decision maker (DM) on his/her preference. On the other hand, to reveal a certain trade-off relation among the conflicting objectives and to serve useful information on trade-off is called as multi-objective analysis (Psarras et al., 2090; Bennett, 1989; Sophos et al., 1980).In our opinion (Shimizu et al., 2007) , therefore, it is proper to say MOEA as a multi-objective analysis method. However, a variety of MOEAs have been developed under the name of "optimization". Regardless of such fundamental definition, MOEA is still useful for MOP since it can derive Pareto front as the essence of trade-off by a single run of computation.

2.2. Multi-objective optimization in terms of scalarlization

As a widely used approach for MOP, certain scalarlized methods have been applied traditionally due to their simplicity in application. The basic idea is to integrate a vector objective function into a scalar one as $V(f(x))$ and transform the original problem as follow.

$$\text{[Problem3]}\ \ \min V(\boldsymbol{f}(\boldsymbol{x})) \text{ subject to } \boldsymbol{x} \in X$$

Its specific formulation is known as the weighting method and given as follows.

$$\text{[Problem4]}\ \ \min \sum_{i=1}^{N} w_i f_i(\boldsymbol{x}) \text{ subject to } \boldsymbol{x} \in X$$

where w_i denotes the weighting coefficient representing the relataive importance of i-th objective function.

On the other hand, another one known as ϵ-constraint method is given as follow.

[Problem5] $\min f_i(\boldsymbol{x})$

$$\text{subject to} \begin{cases} \boldsymbol{x} \in X \\ f_j(\boldsymbol{x}) \leq \epsilon_j, j = 1, \ldots, N, \neq i \end{cases}$$

where ϵ_j is the upper bound compromise for j-th objective function.

Here, it should be noted there never exist general ways to appropriately decide these preference parameters like w_i and ϵ_j beforehand. This is an inherent weakness of these traditional approaches.

3. Proposed Idea and Its Cool Application

3.1. Basic idea to solve SOP by MOEA

The proposed procedure to solve [Problem 1] by MOEA is very simple and deployed in terms of the following propositions.

⟨**Proposition1**⟩ Objectives " $\min f(\boldsymbol{x})$" and " $\max f(\boldsymbol{x})$" always conflict with each other.

⟨**Proposition2**⟩

"[Problem 1'] $\min\{f(\boldsymbol{x}), -f(\boldsymbol{x})\}$ $s.t.$ $\boldsymbol{x} \in X$ " or

"[Problem 1"] $\min\{f(\boldsymbol{x}), 1/f(\boldsymbol{x})\}$ $s.t.$ $\boldsymbol{x} \in X$ " is viewed as a bi-objective problem.

Accordingly, we can generally solve SOP by applying MOEA as follows.

(Step 1) Solve [Problem 1'] (minus) or [Problem 1"] (inverse) by applying a certain MOEA.

(Step 2) Sort the above result in ascending order on objective value $f(\boldsymbol{x})$.

(Step 3) Select the top of the list as the optimal solution for the original problem, [Problem 1].

Visually, the optimal solution will locate at the edge of the Pareto front of [Problem 1'] after stretching the straight front along with generation (Refer to Figure 1 in 4.1).

Regarding the approaches associated with this topic, a few ideas are proposed for the constrained SOPs. The first one (Wang et al., 2007)proposed a scheme that transforms the original problem into an unconstrained bi-objective problem by considering a measure of the constraint violations as the second objective. Another one (Coello, 2000)tries to transform the problem into an unconstrained MOP having the original objective function and its constraints as separate objectives. In this case, the constrained SOP is converted into a MOP with N objectives when the number of constraints is $N - 1$.

It should be noted, in these approaches, trade-off is to be considered between the optimality and the feasibility. Hence, it becomes quite hard to derive some feasible solutions efficiently if any particular ideas could not be introduced in the algorithm. Accordingly, we cannot directly apply any conventional MOEAs to solve the problem. Moreover, the second approach refers likely to many-objective problem more difficult to solve since N becomes large for practical applications. Against these defects, our idea can cope both with the unconstrained and constrained SOPs by just applying the usual MOEA. In other words, we can solve those SOPs even if we have not any EA solver or no knowledge about its usage. To the best of our knowledge, such idea is not proposed anywhere so far.

3.2. Post-optimal evolution for scalarlized MOP

Noticing the above basic idea, we can now solve the scalarlized MOP mentioned in the previous section. This is a cool application of MOEA to extend its availability also not noted anywhere. Actually, this is taken place through applying MOEA to each bi-objective problem as follows.

$$[\text{Problem3}'] \ \min\{V(\boldsymbol{f}(\boldsymbol{x})), -V(\boldsymbol{f}(\boldsymbol{x}))\}$$
$$\text{subject to } \boldsymbol{x} \in X$$

$$[\text{Problem4}'] \ \min\{\Sigma_{i=1}^{N} w_i f_i(\boldsymbol{x}), -\Sigma_{i=1}^{N} w_i f_i(\boldsymbol{x})\}$$
$$\text{subject to } \boldsymbol{x} \in X$$

[Problem5'] $\min\{f_i(\boldsymbol{x}), -f_i(\boldsymbol{x})\}$

$$\text{subject to} \begin{cases} \boldsymbol{x} \in X \\ f_j(\boldsymbol{x}) \leq \epsilon_j, j = 1, \ldots, N, \neq i \end{cases}$$

Moreover, it is possible to cope with the inherent weakness embedded in the scalarlized MOP through a post-optimal evolution mentioned below. For this purpose, our elite-induced MOEA (EI-MOEA) (Shimizu et al., 2012; Shimizu and Nakamura, 2015)is just amenable.

The principle behind EI-MOEA is just simple and straightforward from the original MOEA. Instead of using all randomly generated initial solutions, it introduces some number of the elite solutions that are obtained from a certain procedure. We can expect the existence of elite solutions induce the Pareto front at the direction toward their pre-existing locations. By adjusting the number of such elites, it is able to manipulate a distribution of final solutions so that the result would lie on a specific region on Pareto front. Due to the existence of the elites, selection pressure that might contribute to the accuracy and convergence speed (Deb and Saxena, 2005)is always kept at high level and makes EI-MOEA powerful and computation load smaller.

The aim of the post-optimal evolution concerned here is to re-evaluate the optimal solution obtained at the first round (prior solution) (Shimizu et al., 2016).Hence it is adequate to limit the distribution since it is enough to inspect the solutions just near the prior solution. Actually, this procedure associated with EI-MOEA is taken place as follows.

(Step 1) Solve [Problem 3'] or [Problem 4'] or [Problem 5'] through the procedure mentioned in the previous section.

(Step 2) Select the elite solutions from the above result.

(Step 3) Solve [Problem 6] by applying the same MOEA as (Step 1) after incorporating the elites into a set of random initial solutions.

$$\text{[Problem6]} \ \min \boldsymbol{f}(\boldsymbol{x}) = \{f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_N(\boldsymbol{x})\}$$
$$\text{subject to} \begin{cases} \boldsymbol{x} \in X \\ \Sigma_{i=1}^{N}(1 - f_i(\boldsymbol{x})/f_i^*)^2 \leq \delta \end{cases}$$

where f_i^* denotes the optimal value of i-th objective function and δ an upper bound that will control the extent of the post-optimal evolution or number of candidates for the final decision. This is closely related to the previously mentioned concern for introducing DM's preference into multi-objective analysis.

To obtain a properly distributed Pareto front within the specified region, we apply NSGA-II (Deb, 2000)for the above problem. NSGA-II uses the idea of elitism that can avoid both deleting the superior solutions found previously and crowding to maintain the diversity of solutions. By virtue of these operators, NSGA-II is presently considered to give very good performance for various problems.

4. Numerical Experiments

4.1. Preliminary evaluation of basic idea

To examine a property of the proposed procedure mentioned in 3.1, we tried to solve SOPs having each objective function of the benchmark problem known as FES1 (Huband et al., 2006; Cortezm, 2014).

$$\min\{f_1(\boldsymbol{x}) = \sum_{i=1}^{D} |x_i - exp(i/D))/3|^{0.5},$$
$$f_2(\boldsymbol{x}) = \sum_{i=1}^{D}(x_i - 0.5cos(10\pi/D) - 0.5)^2)\}$$
$$x_i \in [0, 1], \ i = 1, \ldots, D = 8$$

Actually, we solved the problems by using the open code of NSGA-II in R library with the default tuning parameters (Refer to the library named "mco"), i.e., crossover probability = 0.7 and mutation probability= 0.2. We set up the population size (popsz) and number of generation (gener) as 20 and 200, respectively and showed these results in 1 where the left-hand side graph correspond to the first objective while the right-hand side the second one. We also described the frontier at earlier generation (gener=10) as a smaller graph in the respective figure. We can observe the linear front expanding in the left-top direction.

To examine the performance of these results, we also solved those problems by Nelder-Mead (N-M) method which is a mathematical programming method known as powerful. Here, N-M was applied under the initial values given by rounding the above NSGA-II results at the first decimal point and 500 iterations. Through such search starting near the optimum and under a large number of iteration, we can expect to obtain an almost optimal solution by this search. On the

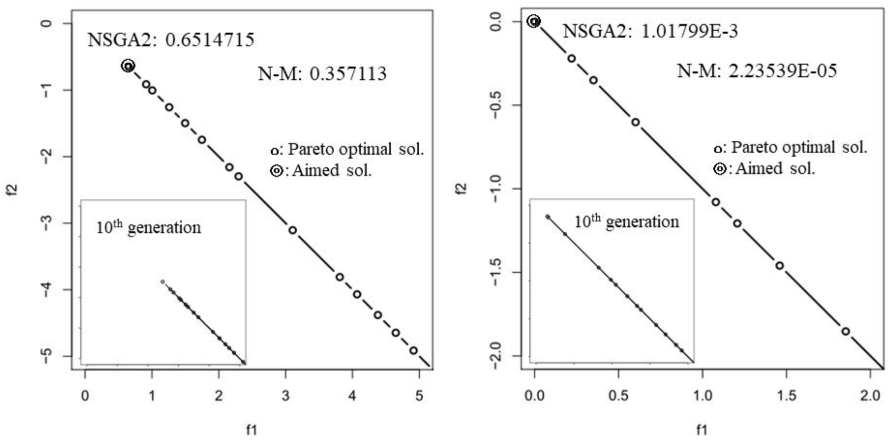

Fig. 1 Features of Pareto front obtained by the propose procedure (NSGA2) for bench-mark problem FES1: Left: Result for the first objective function (min{ $f_1, -f_1$ }), Right: for the second one (min{ $-f_2, f_2$ }) Frontier at earlier generation (gener=10) in small figure expands to the left-top point along with the line.

upper right side of each figure, we put each converged objective function value next to the method name. Knowing the results of NSGA2 are comparable to that of N-M, we can assert SOPs are effectively solvable by MOEA like NSGA-II.

4.2. Evaluation of basic idea with various benchmark problems

Now, to evaluate a comprehensive solution ability through comparison with other methods, we have tuned our interest to the traditionally used ten benchmark problems some of which have multiple peaks of objective functions (Refer to Appendix). We consider one direct search N-M taken in the previous section and four popular EAs such as DE (Differentially Evolution), PSO (Particle Swarm Optimization), GA (Genetic Algorithm) and SA (Simulated Annealing) for this comparison (Refer to the textbook (Shimizu et al., 2007)about their explanations). As same as the earlier section, NSGA-II was selected to apply the proposed basic idea described in 3.1.

Each problem was solved 31 times using open codes all in R library (default tuning parameters were used so that everyone can easily make a double check) ∗2. Regarding the population size (popsz) and generation time (gener), we set those values depending on the dimension of decision variables D as Eqs.(1) and (2), respectively. We show the results of minimum, median, mean and maximum values of objective function in Tables 1-10. Moreover, depending on the known optimal value f_{opt}, we added the success and path numbers as evaluated by Eqs.(3) and (4), respectively.

$$popsz = \min(4(10D/4), 60) \tag{1}$$

$$gener = \min(100 popsz^{0.7}, 200) \tag{2}$$

$$\begin{gathered} Success\# :\ if\ |f_{opt} - f(\boldsymbol{x})| \le eps(1 + |f_{opt}|), \\ then\ count\ Success\#\ when\ eps = 5e^{-3} \end{gathered} \tag{3}$$

$$\begin{gathered} Path\# :\ if\ |f_{opt} - f(\boldsymbol{x})| \le eps(1 + |f_{opt}|), \\ then\ count\ Path\#\ when\ eps = 1e^{-2} \end{gathered} \tag{4}$$

Under the present conditions, we know only DE could get a full mark and only two problems (De Jong & Martin/ Gaddy) are solved correctly by all methods. Being inferior to DE and PSO, the proposed approach outperforms the relative method like GA and the rests. Compared with EAs applying the multi-start search, performance of single-start search like SA was poor overall. On the other hand, N-M has a favorable feature especially for single-peak problems regardless of its simple algorithm.

According to the Success# first and Path# second, we ranked the method as shown in Table 11. Thereat, six apparently poor results of NSGA-II compared with others are analyzed more in detail. We showed their histogram of the objective

∗2 Library or code names for those are as follows, respectively: DE, ”DEoptim”; PSO, ”pso”; GA, ”rbga”; SA & N-M, ”optim”.

Table 1 Comparison among the methods for Shekel's fox hole (D=2, popsz=20, gener=814, f_{opt}=0.9980038)

Item	DE	PSO	GA	N-M	SA	NSGA2
Minimum	0.9980038	0.9980038	0.9980038	0.9980039	0.9980038	0.9980038
Median	0.9980038	0.9980038	0.9980038	10.76318	15.50382	0.9980038
Mean	0.9980038	1.062135	0.9980038	11.89891	75.26274	0.9980038
Maximum	0.9980038	1.992031	0.9980039	23.80944	499.9823	0.9980038
Success#	31	29	31	2	1	31
Path#	31	29	31	2	1	31

Table 2 Comparison among the methods for Schwefel (D=2, popsz=20, gener=814, f_{opt}=0.0)

Item	DE	PSO	GA	N-M	SA	NSGA2
Minimum	2.55E-05	2.55E-05	0.00097146	-138.1764	-138.1762	2.57E-058
Median	2.55E-05	2.55E-05	0.01505475	335.578	236.8771	0.001377
Mean	2.55E-05	15.28239	0.01922313	330.5145	294.2544	34.38634
Maximum	2.55E-05	118.4384	0.05370163	691.0067	710.6959	118.4405
Success#	31	27	4	2	3	22
Path#	31	27	9	2	3	22

Table 3 Comparison among the methods for De Jong (D=2, popsz=20, gener=814, f_{opt}=3905.93)

Item	DE	PSO	GA	N-M	SA	NSGA2
Minimum	3905.93	3905.93	3905.93	3905.93	3905.93	3905.93
Median	3905.93	3905.93	3905.93	3905.93	3905.93	3905.93
Mean	3905.93	3905.93	3905.95	3905.96	3905.93	3905.94
Maximum	3905.93	3905.93	3905.99	3906.69	3905.93	3905.96
Success#	31	31	31	31	31	31
Path#	31	31	31	31	31	31

Table 4 Comparison among the methods for Goldstein/Price (D=2, popsz=20, gener=814, f_{opt}=3.0)

Item	DE	PSO	GA	N-M	SA	NSGA2
Minimum	3.0	3.0	3.00007	3.0	3.00005	3.00006
Median	3.0	3.0	3.00071	3.00114	3.00054	3.00076
Mean	3.0	3.0	3.00080	78.77431	10.83976	3.00707
Maximum	3.0	3.0	3.00179	840.0000	84.00488	3.17030
Success#	31	31	31	16	28	30
Path#	31	31	31	16	28	30

Table 5 Comparison among the methods for Branin (D=2, popsz=20, gener=814, f_{opt}=0.397727)

Item	DE	PSO	GA	N-M	SA	NSGA2
Minimum	0.397727	0.397727	0.397728	0.397727	0.397728	0.397728
Median	0.397727	0.397727	0.397774	0.397727	0.397795	0.397752
Mean	0.397727	0.397727	0.397775	0.397728	0.397852	0.474839
Maximum	0.397727	0.397727	0.397854	0.397728	0.398219	2.78600
Success#	31	31	31	31	31	30
Path#	31	31	31	31	31	30

Table 6 Comparison among the methods for Martin/ Gaddy (D=2, popsz=20, gener=814, f_{opt}=0.0)

Item	DE	PSO	GA	N-M	SA	NSGA2
Minimum	0.0	0.0	1.80E-06	0.0	4.14E-06	4.0E-07
Median	0.0	0.0	1.65E-05	0.0	5.47E-05	2.17E-05
Mean	0.0	0.0	1.86E-05	0.0	7.30E-05	4.67E-05
Maximum	0.0	0.0	4.22E-05	3.85E-07	0.000230	0.000287
Success#	31	31	31	31	31	31
Path#	31	31	31	31	31	31

Table 7 Comparison among the methods for Rosenbrock (D=2, popsz=20, gener=814, f_{opt}=0.0)

Item	DE	PSO	GA	N-M	SA	NSGA2
Minimum	0.0	0.0	5.19E-07	0.0	2.96E-06	1.0E-07
Median	0.0	0.0	0.004154	6.95E-07	0.000172	0.002594
Mean	0.0	0.0	0.014955	2.00E-06	0.000342	0.011054
Maximum	0.0	0.0	0.065285	1.53E-05	0.001362	0.126944
Success#	31	31	16	31	31	20
Path#	31	31	16	31	31	23

Table 8 Comparison among the methods for 4D-Rosenbrock (D=4, popsz=60, gener=1322, f_{opt}=0.0)

Item	DE	PSO	GA	N-M	SA	NSGA2
Minimum	0.0	0.0001082	0.009900	3.04E-07	0.006863	0.00037
Median	0.0	0.0007814	0.601918	1.48E-05	0.029538	0.65946
Mean	0.0	0.0009015	0.578091	0.716831	0.029936	1.00236
Maximum	0.0	0.0022440	1.243803	3.707390	0.080686	3.82302
Success#	31	31	0	25	0	2
Path#	31	31	1	25	2	2

Table 9 Comparison among the methods for Hyper sphere (D=6, popsz=60, gener=1756, f_{opt}=0.0)

Item	DE	PSO	GA	N-M	SA	NSGA2
Minimum	0.0	0.0	5.17E-06	3.02E-07	0.006899	3.59E-05
Median	0.0	0.0	1.76E-05	3.97E-06	0.036040	0.000690
Mean	0.0	0.0	1.69E-05	7.55E-05	0.036648	0.000964
Maximum	0.0	0.0	2.64E-05	0.000921	0.057633	0.007309
Success#	31	31	31	31	0	30
Path#	31	31	31	31	1	31

Table 10 Comparison among the methods for Griewangk (D=10, popsz=60, gener=1756, f_{opt}=0.0)

Item	DE	PSO	GA	N-M	SA	NSGA2
Minimum	0.0	0.0	1.02E-06	0.002927	0.659215	1.00E-04
Median	0.0	0.0	0.007397	0.033673	0.888272	0.007543
Mean	0.0	0.000958	0.004536	0.074354	0.873531	0.010779
Maximum	0.0	0.007396	0.007402	0.367000	0.990362	0.043669
Success#	31	27	12	1	0	10
Path#	31	31	31	3	0	26

Table 11 Rank of method for each benchmark problem on Success# and average success rate over all

Problem	1st	2nd	3rd	4th	5th	last
Shekel's fox hole	DE, GA, **NSGA2**			PSO	N-M	SA
Schwefel	DE	PSO	**NSGA2**	GA	SA	N-M
De Jong	*All solved successfully*					
Goldstein/Price	DE, PSO, GA			**NSGA2**	SA	N-M
Branin	DE, PSO, GA, N-M, SA					**NSGA2**
Martin/ Gaddy	*All solved successfully*					
Rosenbrock	DE, PSO, SA, N-M				**NSGA2**	GA
4-D Rosenbrock	DE, PSO		N-M	**NSGA2**	SA	GA
Hyper sphere	DE, PSO,GA,N-M				**NSGA2**	SA
Griewangk	DE	PSO	GA	**NSGA2**	N-M	SA
Average rate of success [%]	DE=100	PSO= 97.8	**NSGA2=** 81.4	GA= 73.8	N-M= 71.7	SA= 55.9

value in 2. The feature reveals the proposed approach almost attains at a satisfactory level except for 4D-Rosenbrock and Griewangk problems. This is because the frequency of the best range is the highest and degradations stay a little along the range except for only a few cases. Hence, we can conclude the total performance ranks at the third place following DE and PSO a bit behind.

As shown at the bottom line of Table 11, such assertion is also supported from the average rate of success over all benchmark problems. Moreover, according to similar evaluation done elsewhere (Shimizu, 2011)for those benchmark problems under Fortran programming code, DE performed well (average success rate = 98.7%) just as the above results. Meanwhile the poor performance of PSO (average success rate = 48.7%) thereat might arise the evaluation of NSGA-II somehow in this study. From all of these, we can finally claim the proposed approach is satisfactorily efficient as a solution method for SOPs.

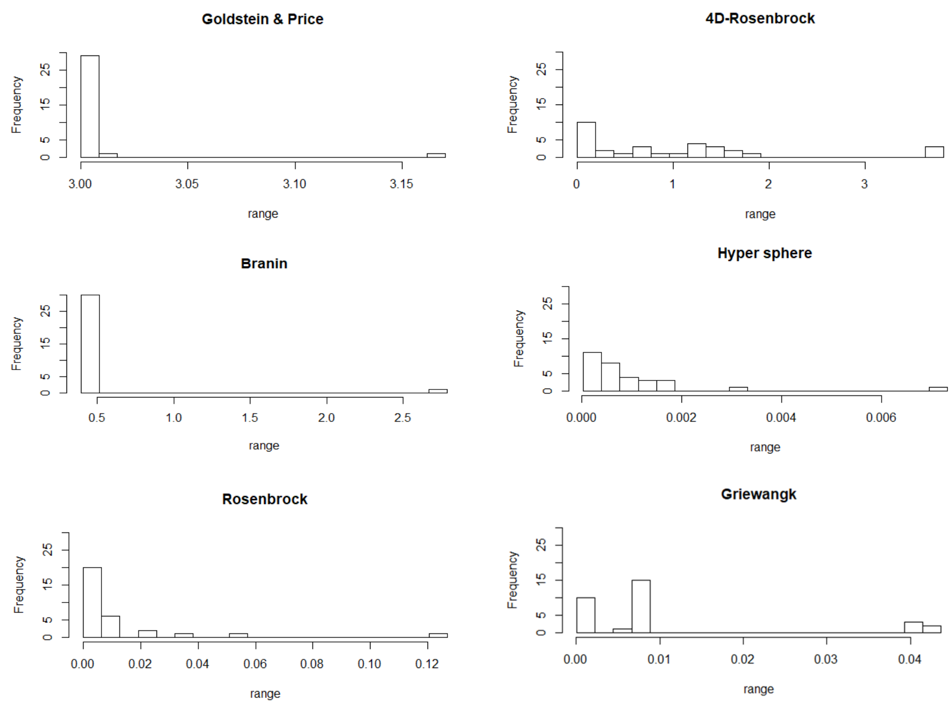

Fig. 2 Histogram of success number for lower rank results regarding NSGA2 (Range refers to f_{opt}).

4.3. Post-optimal evolution for scalarlized MOP

As mentioned already, though the weighting and $\epsilon-$constraint methods have been used due to the effectiveness regardless of their simplicity, a defect of those methods refers to the stiff setting of preference parameters like weighting and ϵ-constraint values. In fact, it is almost impossible to pre-determine those values appropriately according to DM's preference. Hence, it makes sense to re-evaluate the result after such prior optimization through the post-optimal evolution following the procedure mentioned in 3.2.

To discuss on this matter, we first transformed the multi-objective FES1 benchmark into SOPs in two ways, i.e., [Problem 4] and [Problem 5]. Then, we supposed three decision environments in terms of the preference parameters, i.e., case $\epsilon_2 = 0.1$ weighs on f_2 and $\epsilon_2 = 0.7$ on f_1 while case "Weighting" balances both. Finally, solving [Problem 4'] or [Problem 5'] by NGSA-II, we obtained each result shown in "Prior" column of Table 12.

In the next step, we worked on the post-optimal evolution ([Problem 6]) for each case. Using the elite-induced NSGA-II under the conditions popsz=6, gener=200 and introducing a single elite shown in "Prior" column, we obtained the respective result as shown in "Post-optimal" column. Since the population size 6 is small enough for the final decision, we neglected to limit the distribution, i.e., $\delta = \infty$.

Among them, we used bold face to show the best solution while underline infeasible ones. By virtue of the post-optimal evolution, let us note some solutions shown by *italic letters* outperform the prior solution. Hence, we could find out more favorable solution by elaborately inspecting those solutions again. In fact, since the pre-determined ϵ-constraint values are not strictly definite, it is meaningful to work on this re-evaluation over the underlined infeasible solutions. For example, as seen in the case of $\epsilon_2 = 0.1$, an infeasible (2.5548, 0.1086) might be more preferable to the best (2.7178, 0.0875) or the elite (2.6366, 0.1) since allowing a slight violation on f_2 can give a big return on f_1.

Figure 3 describes a summary of results for the original optimizations (w12, $\epsilon = 0.1$ & $\epsilon = 0.7$) and the post-optimal evolutions (post-w12, post<0.1 & post<0.7). (In reference, we add the overall Pareto front derived by ordinal NSGA-II under the conditions popsz=20, gener=100.) We know each post-optimal evolution is possible to derive the particular segment covering the proper Pareto front around the respective prior optimal solution. Particularly, we should note this is done under a small population size such as six by using the elite induced version of NSGA-II. Through inspecting such front where DM must be most interested in, we can get a chance to re-consider the prior decision made under the pre-determined indefinite preference parameters and improve the quality of multi-objective optimization, after all. Eventually, we can relax the defect of classical scalarlized methods and renew them as more adaptive approaches.

Table 12 Result of FES1 benchmark by the proposed procedure mentioned in 3.2

Employed Method	Preference parameter	Prior ([Problem 4' or 5'])		Post-optimal ([Problem 6])	
		(f_1, f_2): Elite	Objective value	(f_1, f_2)	Objective value
Weighting	$w = (0.2, 0.8)$	(1.8510, 0.2077)	0.5363	***(1.4875, 0.2089)***	***0.4646***
				(1.2923, 0.2749)	*0.4784*
				(1.7935, 0.1795)	*0.5023*
				(2.3434, 0.1357)	0.5772
				(2.3603, 0.1347)	0.5798
$\epsilon-$ constraint	$\epsilon_2 = 0.1$	(2.6366, 0.1)	2.6366	(2.2045, 0.1406)	2.2045
				(2.4921, 0.1168)	2.4921
				(2.5548, 0.1086)	2.5548
				(2.7178, 0.0875)	**2.7178**
				(2.9363, 0.0738)	2.9363
				(3.0837, 0.0588)	3.0837
	$\epsilon_2 = 0.7$	(1.0531, 0.7)	1.0531	(0.5971, 0.7773)	0.5971
				(0.7454, 0.7163)	0.7454
				(0.8292, 0.5473)	***0.8292***
				(0.9483, 0.4234)	*0.9483*
				(1.1198, 0.3913)	1.1198

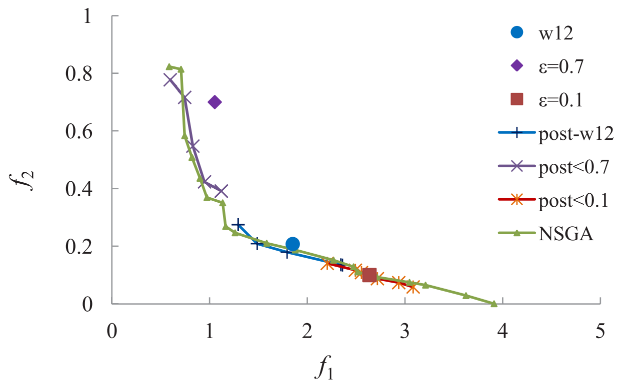

Fig. 3 Post-optimal evolution for three optimal solutions obtained by classical methods.

5. Conclusion

To make a rational decision for difficult problems in various fields, multi-objective evolutionary algorithms are being interested in these decades. Though they are useful techniques for multi-objective analysis, in this paper, we have proposed a simple procedure for solving single-objective optimization problems by them. It provides a new application of them to enhance the availability. Moreover, the idea has been deployed as a post-optimal evolution to repair the shortcomings inherent to the classic multi-objective optimization methods like weighing and ϵ-constraint approaches. Actually, it is developed in co-operation with our elite-induced evolutionary algorithms. After preliminarily examining a few properties of the idea, a set of benchmark problems have been solved by NSGA-II and verified the effectiveness of the proposed idea. Moreover, in terms of the proposed post-optimal evolution, we can engage more practically and flexibly in decision-making encountered today and in future as well.

Comparison with the inverse formulation ([Problem 1”]) is left for further investigation. Moreover, it is interesting to apply the idea to modern scalarlized methods like $\mathrm{MOON}^2/\mathrm{MOON}^{2R}$ (Shimizu et al., 2004)(Finally, refer to [Problem 3’]). To compare the performance among the other MOEAs like MODE, MOPSO, etc. also seems to be meaningful.

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1. Benchmark problems with multi-peak

Outline of the benchmark problems is listed below as problem name, objective function, range and global optimum values.

Shekel: $1/f(x) = 0.002 + \Sigma_{j=1}^{25} \frac{1}{j+\Sigma_{i=1}^{2}(x_i - a_{ij})^6}$,
$a_{ij} = (-32, -16, 0, 16, 32, -32, -16, 0, 16, 32, -32, -16, 0,$
$16, 32, -32, -16, 0, 16, 32, -32, -16, 0, 16, 32;$
$-32, -32, -32, -32, -32, -16, -16, -16, -16, -16, 0, 0,$
$0, 0, 0, 16, 16, 16, 16, 16, 32, 32, 32, 32, 32)$
$-50 \leq x_i \leq 50$
$x = (-32.0, -32.0),\ f_{opt} = 0.998004$

Schwefel: $f(x) = 418.9829 - \Sigma_{i=1}^{2} x_i \sin(\sqrt{|x_i|})$,
$-500 \leq x_i \leq 500$
$x = (418.9829, 418.9829),\ f_{opt} = 0$

De Jong: $f(x) = 3905.93 + 100(x_1^2 - x_2)^2 + (1 - x_1)^2$,
$-2 \leq x_i \leq 2$
$x = (1, 1),\ f_{opt} = 3905.93$

Goldstein & Price: $f(x) = \{1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1x_2 + 3x_2^2)\}\{30 + (2x_1 - 3x_2)^2(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)\}$,
$-2 \leq x_i \leq 2$
$x = (0, -1),\ f_{opt} = 3$

Branin: $f(x) = a(x_2 - bx_1^2 + cx_1 - d)^2 + e(1 - f)\cos(x_1) + e$,
$a = 1,\ b = \frac{5.1}{4}(\frac{7}{22})^2,\ c = \frac{35}{22},\ d = 6,\ e = 10,\ f = \frac{7}{176}$
$-5 \leq x_i \leq 10$
$x = (-\frac{22}{7}, 12.275),\ x = (\frac{22}{7}, 2.275),\ x = (\frac{66}{7}, 2.475)$,
$f_{opt} = 0.3977272$

Martin & Gaddy: $f(x) = (x_1 - x_2)^2 + \{(x_1 + x_2 - 10)/3\}^2$,
$0 \leq x_i \leq 10$
$x = (5, 5),\ f_{opt} = 0$

Rosenbrock: $f(x) = 100(x_1^2 - x_2)^2 + (1 - x_1)^2$,
$-2 \leq x_i \leq 2$
$x = (1, 1),\ f_{opt} = 0$

4D-Rosenbrock: $f(x) = \Sigma_{i=1}^{3}\{100(x_i^2 - x_{i+1})^2 + (1 - x_i)^2\}$,
$-2 \leq x_i \leq 2$
$x = (1, 1, 1, 1),\ f_{opt} = 0$

Hyper sphere: $f(x) = \Sigma_{i=1}^{6} x_i^2$,
$-6 \leq x_i \leq 6$
$x = (0, \cdots, 0),\ f_{opt} = 0$

Griewangk: $f(x) = 1 + \Sigma_{i=1}^{10} \frac{x_i^2}{4000} - \Pi_{i=1}^{10} \cos(\frac{x_i}{\sqrt{i}})$,
$-5 \leq x_i \leq 5$
$x = (0, 0, \cdots, 0, 0),\ f_{opt} = 0$